

7.4 Comparing and Ordering Irrational Numbers on a Number Line

To compare irrational numbers that are square roots, we can simply examine the number that we are taking the square root of. For example, we know that $\sqrt{15} < \sqrt{17}$ because 15 is less than 17.

However, when we compare irrational numbers such as $\sqrt{10}$ and π , it is simplest to compare rational approximations of each written as a decimal. We know that $\sqrt{10} \approx 3.16$ and that $\pi \approx 3.14$. Therefore we can say that $\sqrt{10} > \pi$. Notice that it was useful to approximate the irrational numbers to two decimal places in this case even though it wasn't entirely necessary.

The same is true for comparing irrational and rational numbers. By finding a rational approximation of the irrational numbers, we can compare values such as π and $\frac{22}{7}$. For these numbers we may have to go to a three decimal places for our approximation and the use of a calculator would make sense. Rounded to three decimal places, we find that $\pi \approx 3.142$ and $\frac{22}{7} \approx 3.143$ which means that $\pi < \frac{22}{7}$.

Once we know how to compare two numbers, we can then order a set of numbers through comparison of two numbers at a time. For example, we could list from least to greatest $\sqrt{10}$, π , 3.14, and $\frac{22}{7}$. We know the following:

$$\sqrt{10} \approx 3.162$$

$$\pi \approx 3.142$$

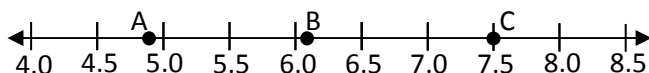
$$\frac{22}{7} \approx 3.143$$

We see that $\sqrt{10}$ is greater than all the other numbers given. We also note that 3.14 is the smallest because it is equal to 3.140 to three decimal places. Therefore we can list them in order like so: 3.14, π , $\frac{22}{7}$, $\sqrt{10}$. Notice that the closer the numbers are to each other, the more decimal places of accuracy we need in our rational approximation.

Locating Irrational Numbers on a Number Line

Again, rational approximations of irrational numbers will be our friend. On a number line, we generally list rational number markers. On the simplest number lines, we count by integers. On a standard English ruler, we count by fractions, usually $\frac{1}{16}$ inch or $\frac{1}{8}$ inch. On a standard metric ruler, we count by millimeters which are each .1 centimeters. No matter how the number line is set up, we will still need the rational approximations of the irrational numbers.

For example, let's try to place the following irrational numbers on the number line: $\sqrt{37}$, $\sqrt{42}$, and $\sqrt{24}$. First we will make a quick, one decimal place approximation of each. $\sqrt{37} \approx 6.1$ since 37 is just over 36, $\sqrt{56} \approx 7.5$ since 56 is about half-way between the perfect squares of 49 and 64, and $\sqrt{24} \approx 4.9$ since 24 is just under 25. Now examine where the dots are located on the following number line.



Note that point A must be $\sqrt{24}$ since it is just under 5, point B must be $\sqrt{37}$ since it is just over 6, and point C must be $\sqrt{56}$ since it is right at 7.5 on the number line.

In the same way that you can identify which point on a number line goes with which irrational number, you can also place points on a number line to represent the irrational number.

Lesson 7.4

Place a point on the number line given for each of the following irrational numbers.

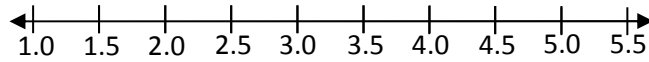
1. Point A: $\sqrt{2}$

2. Point B: $\sqrt{17}$

3. Point C: $\sqrt{11}$

4. Point D: $\sqrt{8}$

5. Point E: $\sqrt{5}$



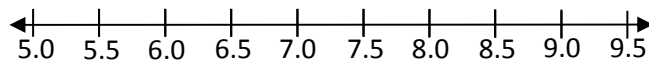
6. Point V: $\sqrt{26}$

7. Point W: $\sqrt{88}$

8. Point X: $\sqrt{77}$

9. Point Y: $\sqrt{37}$

10. Point Z: $\sqrt{30}$



Name the point on the number line associated with each irrational number.

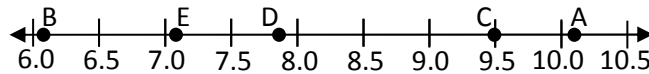
11. $\sqrt{50}$

12. $\sqrt{103}$

13. $\sqrt{62}$

14. $\sqrt{90}$

15. $\sqrt{37}$



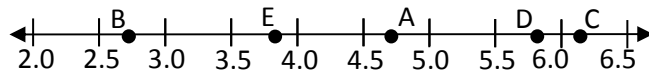
16. $\sqrt{7}$

17. $\sqrt{22}$

18. $\sqrt{34}$

19. $\sqrt{38}$

20. $\sqrt{15}$



Compare the following numbers using < or >.

21. $\sqrt{32}$ 5.1

22. $\sqrt{38}$ $\sqrt{42}$

23. $\sqrt{17}$ $\frac{9}{2}$

24. $\sqrt{49}$ 7.1

25. $\sqrt{99}$ $\frac{28}{3}$

26. $\sqrt{17}$ 4.5

27. $\frac{43}{5}$ $\sqrt{65}$

28. $\sqrt{12}$ $\sqrt{21}$

29. $\sqrt{16}$ 3.9

30. $\sqrt{2}$ $\frac{7}{4}$

31. $\sqrt{50}$ $\frac{15}{2}$

32. $\sqrt{9}$ 3.01

List the following numbers in order from least to greatest.

33. $\sqrt{16}$, 4.2, $\frac{39}{8}$

34. $\sqrt{24}$, $\sqrt{33}$, 5.1

35. $\sqrt{100}$, $\sqrt{110}$, $\frac{32}{7}$

36. 9.4, $\frac{19}{2}$, $\sqrt{80}$

37. $\sqrt{35}$, $\sqrt{32}$, $\sqrt{37}$, $\frac{22}{3}$

38. $\sqrt{10}$, 3.5, $\sqrt{15}$, $\frac{13}{3}$

39. $\sqrt{65}$, $\sqrt{60}$, 8.5, $\frac{37}{4}$

40. $\sqrt{39}$, $\sqrt{25}$, 5.3, $\sqrt{26}$, $\frac{23}{4}$

41. $\sqrt{12}$, $\sqrt{15}$, 4.3, $\sqrt{9}$, $\frac{14}{5}$

42. $\sqrt{49}$, $\sqrt{63}$, 7.3, $\sqrt{38}$, $\frac{15}{2}$